Ball and Blumenfeld Reply: We are grateful for Marder's Comment [1] on our Letter [2], and explain below why the numerical data presented by him do not contradict our conclusions. The essential point is that the logarithmic stress oscillations we have found are an exact result and solve the field equations with the proper boundary conditions. However, we do agree that they are difficult to observe numerically. Whether these oscillations bias the pattern formation is a question not addressed by Marder's simulations, but on which we will remark. We also explain the unaccounted for phenomena that he observes, namely, nearly linear oscillations in the simulations and the power-law decay of the curvature of the simulated tip with the order of the approximation.

Using the elegant method of Muskhelishvili [1], Marder searches for our predicted oscillations in $\ln r$ by approaching the tip of the wedge along a ray [3]. The general form of the oscillations we found is

$$r^{\mu+i\nu-1}\cos[(\mu+i\nu\pm 1)\theta]\,,$$

which increases in magnitude roughly as $e^{\nu\theta}$ as a function of angle θ relative to the direction forward from the wedge axis. For the case Marder studies, $\alpha=3\pi/4$, we find that the leading oscillation is $\nu\approx 1.1$ and therefore expect the period to be of order $2\pi/\nu\approx 5.7$. So the first difficulty in the observability of our oscillations lies in the fact that although numerically impressive, the range of useful data ($\ln r$ between -9 and -4) is too short to detect an oscillatory trend. Second, this difficulty is magnified due to the appearance of oscillations in r, whose origin is geometrical as we show below. A possible hint for the existence of our solutions could actually be found in the former version of Marder's Comment, where the approach was along the wedge axis. As we argue below, in this direction the spurious oscillations are damped and indeed an indication of one period of the oscillation (with the right length $2\pi/\nu\approx 6$ in $\ln r$) could be detected.

However, we would like to comment on our results from a different angle: The length of our predicted oscillations increases rapidly with r. This could imply that they may be too long to be significant to existing numerical simulations of rupture of lattices of moderate size [4], although they should be relevant to the onset of scale-invariant sidebranching in real systems that can achieve such sizes.

The periodicity in r that Marder observes stems from his numerical method. In this method one maps the planar wedge (in the z plane) conformally onto the unit circle (in the ω plane) and expands the mapping function in Laurent series in ω^{-n} ($n=1,2,\ldots,N$). The stress field is then calculated exactly for the shape defined by the truncated map. On the unit circle $[\omega(\phi)=\exp(i\phi)]$ the map is a Fourier series in "charge space" (to borrow an electrostatic term), and as usual in Fourier expansion the resolution is limited, $\Delta Q\sim 1/N$. The conformal mapping of the wedge gives $Q\sim r^{\pi/2\alpha}$, leading to $\Delta r\sim(\Delta Q)^{2\alpha/\pi}\sim N^{-2\alpha/\pi}$. In his simulations $2\alpha/\pi=1.5$, which explains exactly the observed decay of the radius curvature as $N^{-1.5}$. Further, this also suggests that in the simulations the edges of the wedge are not straight lines but rather have corrugations oscillatory in $Q/\Delta Q=(r/\Delta r)^{\pi/2\alpha}$. The stress field near the edges must follow these oscillations, which are then observed in Marder's results (periodic in $r^{2/3}$, not r). We expect the amplitude of these spurious oscillations to decrease away from the edges and towards the axis. Unfortunately, the logarithmic oscillations also decrease in that direction as mentioned above.

We did not previously predict the absolute amplitude of the oscillations in $\ln r$, only their form. Qualitatively their amplitude and phase should be determined by the shape with which the tip of the wedge is rounded off. As explained above, in Marder's case this means that these quantities are determined indirectly by the length of his Laurant expansion for the conformal map from the unit circle. We would expect the oscillations to begin with amplitude that is comparable to the leading term in the scale of the tip radius, and then to decline in relative significance away from the tip as $r^{\mu-m_+}$, where m_+ is the main singularity and μ the real part of the exponent for the first oscillatory correction [2]. The oscillations are then expected to be significant just behind the tip, which is precisely where the sidebranching morphology must be nucleated in tip-led growth; sidebranching near the tip can in turn alter the phase and amplitude of the oscillations, which raises the possibility of feedback effects. We therefore stand by our suggestion that the oscillations may be of physical importance. However, for the ideal case of a perfectly sharp and smooth wedge, the absence of scale means that the oscillating corrections to the dominant behavior are damped on an infinitesimally small scale, and as for the case of Ling's wedge of two intersecting circles [1], they should be absent.

Robin C. Ball and Raphael Blumenfeld
Cavendish Laboratory, Cambridge University
Madingley Road
Cambridge CB3 0HE, United Kingdom

Received 4 October 1991
PACS numbers: 62.20.Mk, 46.30.Nz, 64.60.Ak

[1] M. Marder, preceding Comment, Phys. Rev. Lett. **68**, 2253 (1992), and references therein.
[2] Robin C. Ball and Raphael Blumenfeld, Phys. Rev. Lett. **65**, 1784 (1990).
[3] In an earlier version of the Comment, Marder chose the ray along the wedge axis ($\theta=0$). We pointed out that in this direction we expect the magnitude of our predicted oscillations to be minimal and downgraded by a factor of 13 compared to approaching along the edge of the wedge ($\theta=\alpha$). We are grateful to him for redoing his numerical analysis as we suggested.
[4] E.g., P. Meakin, G. Li, L. M. Sander, E. Louis, and F. Guinea, J. Phys. A **22**, 1393 (1989).