Comment on "Nonlinear susceptibilities of granular matter"

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The results of Stroud and Hui (SH) on the conductivity of weakly nonlinear media are shown to imply the scaling behavior of the critical current I_c, which marks the transition from linear to nonlinear behavior. Applying this idea to a specific system, we suggest a possible explanation of experimental results reported previously. The scaling behavior of I_c is also calculated for the case of a perturbed strongly nonlinear system.

In a recent paper, Stroud and Hui[1] (SH) studied the electrical properties of a weakly nonlinear composite medium at the percolation threshold p_c. Assuming a small cubic correction to the local linear electric field, the current density between two terminal plates, positioned a large distance L apart, is

$$\mathbf{J}=[\sigma_e(L)+a_e(L)|\mathbf{E}_0|^2]\mathbf{E}_0 , \tag{1}$$

where $|\mathbf{E}_0|L$ is the potential difference between the terminals, σ_e and a_e are, respectively, the effective linear conductivity and the corresponding effective amplitude of the nonlinear perturbation. For the case $a_e|E_0|^2 \ll \sigma_e$, SH find that

$$a_e \sim L^d \delta\sigma_e^2 , \tag{2}$$

where $\delta\sigma_e^2$ stands for the rms conductivity fluctuations in the unperturbed system. In our Comment we first use this result to derive the scaling properties of the critical current I_c, at which the I-V curve crosses over from linear to nonlinear behaviour. Such a crossover has been studied in thin gold films.[2,3] Relation (2) is the continuum analog of Aharony's result[3] for random resistor networks (RRN's). Our result is used to propose a possible explanation for a reported experimental observation[2] within the framework of percolation theory. We then remark that this result can be generalized to strongly nonlinear RRN's perturbed by a different nonlinear term. Confining ourselves first to a perturbed linear system, we define the effective conductance Σ, the nonlinear conductance A, and the critical exponents t and ζ, via

$$\begin{aligned} \Sigma &= \sigma_e L^{d-2} , \\ \sigma_e &\sim \xi^{-t/\nu} , \\ t &= \zeta+(d-2)\nu , \\ A &= a_e L^{d-4} , \end{aligned} \tag{3}$$

where $\xi \sim |p-p_c|^{-\nu}$ is the percolation correlation length, and we consider systems whose size L is larger than ξ. Rewriting (1) as

$$I=(\Sigma+AV^2)V , \tag{4}$$

we can find the critical current by equating the two terms on the right-hand side (rhs) of (4):

$$V_c \approx (\Sigma/A)^{1/2} , \tag{5}$$

$$S_R \equiv \delta\sigma_e^2/\sigma_e^2 \sim L^{-d}\xi^{\kappa/\nu} , \tag{6}$$

which defines the exponent κ, and using (2), (3), and (5), we obtain

$$I_c \sim L^{d-1}\sigma_e^{(1+\kappa/t)/2} . \tag{7}$$

Now we can use existing estimates for the values of κ (Refs. 4 and 5) and t (Ref. 6) in binary RRN's to find that, in $d=2$, 3, 4, 5, and 6, respectively, $(1+\kappa/t)/2=0.95$, 0.86, 0.83, 0.83, and $\frac{5}{6}$.

However, there are percolation models where this exponent is much larger than any of these values: for example, in two-dimensional random void continuum percolation one can use existing results to deduce $I_c \sim \sigma_e^{2.1}$. Gefen *et al.*[2] found, in a physical two-dimensional percolating system, that this power is 1.47 ± 0.1, which is larger than existing bounds on the universal value in binary RRN's.[2,3] They therefore concluded that this discrepancy may indicate a conduction mechanism other than percolation. However, their system may simply belong to a different universality class of percolation, thus allowing for such a result without having to invoke a new mechanism.

Relation (7) can be easily generalized to strongly nonlinear media perturbed by another nonlinear term

$$J=\sigma_e E^{\alpha_1}+a_e E^{\alpha_2}, \quad \alpha_2>\alpha_1, \quad a_e E^{\alpha_2-\alpha_1} \ll \sigma_e . \tag{8}$$

The exponents $t(\alpha_1)$ and $\zeta(\alpha_1)$ now depend on α_1: they have the same meaning as in the linear case ($\alpha_1=1$), but the relation between them generalizes to

$$t(\alpha_1)=\zeta(\alpha_1)+(d-1-\alpha_1)\nu$$

(Ref. 8). One can define the higher-order cumulants[9] $\delta\sigma_e^q$ and, consequently, the quantity

$$\delta\sigma_e^q/\sigma_e^q \sim L^{d(1-q)}|p-p_c|^{-\kappa(q,\alpha_1)} . \tag{9}$$

For noninteger values of q this quantity can be expressed in terms of moments of the current density as in the discrete case.[9] SH's result (2) can now be generalized to (see also Ref. 3)

$$a_e \sim L^d \delta\sigma_e^{(\alpha_2+1)/(\alpha_1+1)} . \tag{10}$$

Repeating the considerations of Eqs. (4)–(7), we find in this case for the critical current

$$I_c \sim L^{d-1+d\alpha_1[1/(\alpha_1+1)-1/(\alpha_2-\alpha_1)]}\sigma_e^{1/(\alpha_1+1)+\kappa[(\alpha_2+1)/(\alpha_1+1),\alpha_1]\alpha_1/(\alpha_2-\alpha_1)t(\alpha_1)} . \tag{11}$$

So far we have focused on systems larger than ξ. When $L \ll \xi$, we can write

$$\delta\sigma_e^q/\sigma_e^q \sim L^{[\psi(q,\alpha_1)-q\zeta(\alpha_1)]/\nu} ,$$

where ψ characterizes the scaling of the qth cumulant of the global resistance distribution due to local resistance fluctuations[4,5,7,9] [note that $\zeta(\alpha_1)=\psi(1,\alpha_1)$ and $\psi+q(d\nu-\zeta)=\kappa+d\nu$],

$$[\delta R^q]_c \sim L^{\psi(q,\alpha_1)/\nu} . \tag{12}$$

Since σ_e scales as $L^{-t/\nu}$, (11) can be rewritten as a function only of L:

$$I_c \sim L^{d\alpha_1[1/(\alpha_1+1)-1/(\alpha_2-\alpha_1)]+[\zeta/(\alpha_1+1)\nu][\alpha_1(\alpha_2+1)/(\alpha_2-\alpha_1)-1]-[\alpha_1/(\alpha_2-\alpha_1)\nu]\psi[(\alpha_2+1)/(\alpha_1+1),\alpha_1]} . \tag{13}$$

Clearly, if $\alpha_1 > \alpha_2$, the restriction in (8) may break down for small enough fields. In this case, as the system becomes very large, only the small fields (or currents in networks) dominate, and the roles of α_1 and α_2 will be exchanged. Therefore, interchanging α_1 and α_2 in the above results yields the critical exponent for the critical current in these systems.

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