Blumenfeld and Edwards Reply: In our Letter [1], we have introduced a volume function $\mathcal{W}$ that makes it possible to carry out explicit calculations of the configurational entropy of two dimensional assemblies of jammed granular materials. The formalism has been also shown to apply directly to foams and cellular structures.

In the preceding Comment [2], Ciamarra observes that the volume function, as suggested in [1] does not describe the volume of a class of granular systems containing nonconvex cells of particular configurations. The reason is that our basic volume elements, the quadrons, may ill-tessellate some nonconvex voids and so overestimate their volumes.

While in principle correct, the Comment does not invalidate our analysis for several reasons. First, nonconvex cells occur only around ''rattlers'', which are stable only due to the direct action of external or body forces. In the absence of external fields, namely, when loads are applied only through the boundaries, nonconvex cells are unstable and can hardly occur. Therefore, our analysis is rigorously exact for all rattler-free systems and systems loaded through the external boundaries. Second, not all nonconvex voids are ill tessellated. For the systems that Ciamarra has generated, which appear to have an unusually high amount of rattlers, he estimates the fraction of potentially ill-tessellated voids (ITV) between 2.5% and 7% for a wide range of frictions and polydispersity [Fig. (2a) of [2]]. Third, the actual error due to quadrons overlap is a fraction of the volume of the ITV. Therefore, contrary to Ciamarra's estimate, the error that such voids may introduce in the evaluation of $\mathcal{W}$ is very small. This error is estimated below from Ciamarra's data to be less than 0.55%, hardly a ''first crude approximation.'' Fourth, our method and our results in [1] apply rigorously to most cellular materials and foams. This is because these normally do not contain any nonconvex cells.

To estimate the error in $\mathcal{W}$ due to rattlers, we next use Ciamarra's data for his rattler-rich systems. We find that even in these systems the error is much smaller than he suggests. The fraction of ITV, considering all frictions and polydispersity, is between 2.5% and 7%. Following Ciamarra's reasoning, the entire void fraction is between $1 - 0.90 = 0.1$ (for random close packing) and $1 - 0.84 = 0.16$ (for random loose packing). This means that the overall possible volume of ITV is between $0.1 \times 2.5\% = 0.25\%$ and $0.16 \times 7\% = 1.1\%$ of the volume of the system. Additionally, from Ciamarra's Fig. (1), we estimate that quadron overlaps in his shaded ITV gives rise to an overestimate of the volumes with an error that is well below 50%. This means that the total error in the volume of the system due to the ITV is well below 0.55%. This is a much smaller effect than suggested by Ciamarra, and calling it a ''crude approximation'' is unfair.

Nevertheless, Ciamarra's observation is interesting because, ultimately, the issue of rattlers is more relevant to dynamics than to statics. Although neither the Comment nor our original Letter refer to out-of-equilibrium configurations, the above discussion prompted us to think about the extension of the usefulness of $\mathcal{W}$ to such situations. This would be relevant, for example, if one wishes to study the evolution of the volume function under given dynamics through the construction of a master equation. In such an analysis, it would be important to keep in mind that as the packing goes out of equilibrium there is no guarantee that the number of ITV remains small as in statics. Therefore, during the dynamic process, the error in $\mathcal{W}$ may increase and it would drop to the above small value only when the medium comes to a halt and the system resettles into equilibrium. Nevertheless, it is still possible to make use of $\mathcal{W}$ out of equilibrium by adopting the wisdom of conventional thermodynamics and using reversible (or quasiequilibrium) processes. This means that, at every step along the evolution path, the system is kept infinitesimally close to mechanical equilibrium, where $\mathcal{W}$ is a good descriptor. This is an intriguing possibility that remains to be explored. We emphasize, however, that this discussion is downstream from our original Letter, which addressed only the configurational entropy of equilibrium systems.

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