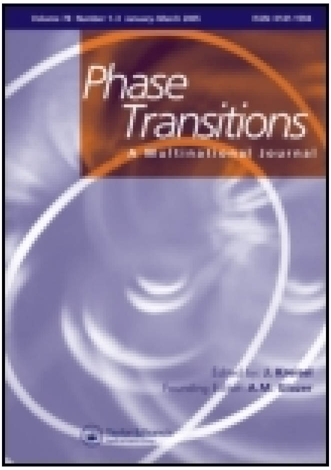

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Raphael Blumenfeld [a]
[a] Cavendish Laboratory , Madingley Rd., Cambridge, CB3 OHE, UK
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HIERARCHICAL STRUCTURE OF DOMAIN WALLS IN MAGNETIC LAYERS

RAPHAEL BLUMENFELD

Cavendish Laboratory, Madingley Rd., Cambridge CB3 OHE, UK

I discuss thin magnetic layers in the context of a two-dimensional ferromagnetic Heisenberg spin system. In the low energy regime it is shown that the system follows a Belavin–Polyakov type of equation. It is argued that unlike in traditional systems where these equations occur only under uniform boundary conditions, the boundary conditions in this case are less restrictive, allowing for a new family of solutions. These solutions consist of magnetic domains of spins that are oriented at relative opposite directions. The boundaries between regions are sharp on the continuous scale but within a domain wall the magnetization changes orientation continuously from one ground state to another. All the magnetic energy in the system is shown to concentrate along the domain walls. It is therefore argued that most favorable for the walls is to rearrange in a hierarchical or fractal fashion because such an arrangement lowers the overall energy density. It is suggested that this hierarchical structure of magnetic domain boundaries should be observable by magnetic force microscopy. Recent results suggest that such configurations may also dominate the structure of domain walls in magnetostrictive materials and magnetic nanotubes.

Keywords: Hierarchical structure; Domain walls; Magnetic layers; Ferromagnetic; Heisenberg spin system; Fractal fashion

1 INTRODUCTION

Two-dimensional and quasi-two-dimensional ferromagnetic systems have a long history as models to study magnetic systems. Recent years saw renewed interest in these systems due to advances in processing and manufacturing technology of magnetic thin films.

Here I report recent progress in the understanding of magnetic domains formation in magnetic layers. The main thrust of this paper is the suggestion that the macroscopic configuration of the domain wall structure should be hierarchical. The theoretical results discussed here bear relation to several other problems where similar mathematical formalism applies. For example, in systems described by the nonlinear sigma model, in the dynamics of spin fluctuations across anti-ferromagnetic spin chains, in the study of geometric phases, in string theory, and in the dynamics of line curves moving in three dimensions, to name a few. The common denominator of all these problems is that the governing equations have the form of the Belavin–Polyakov equations (BPE), with the exception of line curves moving in three dimensions where a more generalized form of these equations is involved [1]. From the practical point of view, these results have applications in magnetic thin films such as Giant Magnetoresistive (GMR) materials for microactuators and microsensors. There also seems to be relevance to the nanotubes technology when these systems are augmented with magnetic properties. This latter application has been suspected following recent results that the theoretical analysis can be made to apply to nonplanar geometries [2].

2 THE SYSTEM

We start from a discrete spin system whose Hamiltonian is

$$H = -J \sum_{\langle mn \rangle} \mathbf{S}_m \cdot \mathbf{S}_n, \tag{2.1}$$

where J is a uniform scalar coupling constant, $\mathbf{S}_n$ is a Heisenberg spin of index n, where the index runs over all possible spins, and the angular brackets denote summation over m and n that are nearest neighbors in real space. For positive values of J the system is ferromagnetic since the lowest energy is when $\mathbf{S}_m$ and $\mathbf{S}_n$ are parallel. The ground state is when all the spins point in the same direction and the entire system is uniformly magnetized. When J is negative the system is antiferro-magnetic and spins favor local antiparallel alignment, in which case it depends on the details of the lattice structure whether the state is ordered or frustrated. Here we focus on two-dimensional ferromagnetic systems. Although the spins are embedded in a two-dimensional plane, the spin vector, $\mathbf{S}$, is three dimensional and is not constrained to point only in the plane.

At low energies above the ground state, the spins are almost parallel locally and the divergence from parallelism is slow over large distances, involving many spins. In this regime one can approximate the discrete system with a continuous model whose equivalent Hamiltonian is

$$H = -J \int |\nabla \mathbf{S}(x, y)|^2 \, \mathrm{d}x \, \mathrm{d}y. \tag{2.2}$$

In 1975, Belavin and Polyakov [3] derived the equation of motion for this Hamiltonian, which reads

$$\mathbf{S}_t = \mathbf{S} \times \nabla^2 \mathbf{S}, \tag{2.3}$$

where the subscript t stands for partial differentiation with respect to time. Here I would like to concentrate on the stationary solutions to (2.3) where the time derivative on the left hand side vanishes

$$\mathbf{S} \times \nabla^2 \mathbf{S} = 0. \tag{2.4}$$

In two dimensions it can be readily shown that solutions to the following equation also solves Eq. (2.4)

$$\mathbf{S}_y = -\mathbf{S} \times \mathbf{S}_x. \tag{2.5}$$

3 THE STRUCTURE OF MAGNETIC DOMAINS

To analyze Eq. (2.5) we recall that the local spin vector is normalized $|\mathbf{S}| = 1$ and write $\mathbf{S}$ in term of its two angular variables

$$\mathbf{S} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \tag{3.1}$$

Substituting the form (3.1) into the vectorial equation (2.5) one obtains two coupled equations for θ and ϕ:

$$\theta_y = -\sin\theta\phi_x; \quad \theta_x = \sin\theta\phi_y. \tag{3.2}$$

The coupled equations in (3.2) are very reminiscent of the so-called BPE, also known as the duality relations, e.g., in the context of antiferromagnetic chains and strings. There is, however, a marked difference: In those systems the BPE are derived only under strictly uniform boundary conditions on θ and ϕ. This restricts the possible solutions to the well known n-instantons. In our case the boundary conditions are not important for obtaining this set of equations and therefore remain arbitrary at this stage. As will be seen below, the lifting of this constraint allows for a new type of kink (i.e., domain wall) solutions which are not available in other systems.

To make progress let us first transform the angle variable θ through

$$\tanh \psi = \cos \theta. \tag{3.3}$$

In the new variables Eqs. (3.2) become:

$$\psi_y = -\phi_x; \quad \psi_x = \phi_y, \tag{3.4}$$

which one recognizes as the Cauchy–Riemman relations (CRR). The boundary conditions on θ transform to boundary conditions on ψ as follows:

$$\theta = (\uparrow, \leftarrow, \downarrow, \rightarrow) = (\pi/2, \pi, -\pi/2, 0) \quad \Rightarrow \quad \psi = (\infty, 0^+, -\infty, 0^-). \tag{3.5}$$

It is convenient to represent the coordinates in complex form $z = x + iy$. Defining then the complex function

$$\Phi = \psi + i\phi, \tag{3.6}$$

the CRR become

$$\psi_{\bar{z}} = i\phi_{\bar{z}} \tag{3.7}$$

and its complex conjugate.

The solutions to the CRR are well known to be all the harmonic functions that contain, at most, simple poles in the complex plane. The poles can be regarded as charges (of both signs). The harmonic functions can be most conveniently presented by expansions in complete sets of functions, such as the Fourier expansion, Legendre polynomials, Bessel functions, etc. The preference of expanding in one set of functions rather than another is determined usually by the boundary conditions or the symmetry of the system. Assuming, for example, a rectangular system it is convenient to use the Fourier transform

$$\begin{aligned}\psi &= \sum_{k=1}^{Q} a_k \cosh(kx)\cos(ky) + \cdots,\\ \phi &= \sum_{k=1}^{Q} a_k \sinh(kx)\sin(ky) + \cdots,\end{aligned} \tag{3.8}$$

where the dots stand for all combinations of hyperbolic and trigonometric functions (e.g., $b_k \cosh(kx)\sin(ky)$, etc.). For later use we recall that the parameter Q is the highest wave number and therefore determines the maximal number of the nodes that the solution can have. This parameter is usually governed by the boundary conditions and the distribution of the charges (which contribute logarithmic terms to Eq. (3.8)).

While these solutions are well known and therefore look uninterestingly innocent for the variables, ψ and ϕ, we have to remember that the physically interesting and relevant fields are θ and ϕ, the two spin angles. Consider then an arbitrary solution for ψ which consists of a set of peaks and troughs in the plane. The function $\tanh\psi$ takes this form and sharpens the transition from every trough to every peak at the position where a node, $\psi=0$, occurs. Away from these locations $\tanh\psi \approx \pm 1$ almost uniformly. Thus $\theta \approx \pm\pi$ on most of the plane and the transition from $+\pi$ to $-\pi$ occurs sharply at $\psi=0$. It follows that the solution describes a configuration of domain walls, located at the contours where $\Psi=0$. The number and the locations of the domain walls are determined by the boundary conditions and by the locations of charges in the system. A positive (negative) point charge is a particular location where the spin is fixed at the up (down) position, and this corresponds to $\psi=\infty(-\infty)$. The number Q is determined by the solution and gives, in turn, the largest number of domain walls that the system may develop.

4 HIERARCHICAL STRUCTURE OF DOMAIN BOUNDARIES

Having established the existence of domain wall solutions and their exact form, let us inspect now the Hamiltonian rewritten in the variables

ψ and ϕ:

$$H = -J \int \frac{|\nabla\psi(x,y)|^2 + |\nabla\phi(x,y)|^2}{\cosh^2 \psi(x,y)} \, dx \, dy. \tag{4.1}$$

This Hamiltonian resembles that of a free particle but with a field dependent mass $1/\cosh^2 \psi$. The denominator obtains its minimum on the lines where $\psi = 0$ ($\cosh^2 \psi = 1$) and it increases exponentially fast as ψ becomes finite of either side of these lines, in which case the integrand in (4.1) is exponentially small. Since $\psi = 0$ only along domain walls then the energy practically vanishes away from the lines that the domain boundaries outline in the plane. It follows that the energy is concentrated along the domain walls and the total energy in the system is proportional to the cumulative length of the domain boundaries, l.

Suppose now that the system's linear size is L so that the system area is $O(L^2)$. The average energy density is

$$\epsilon = \frac{\text{Cumulative length of domain boundaries}}{\text{Total system area}} = \frac{l}{O(L^2)}. \tag{4.2}$$

If the spatial distribution of domain walls in the system is uniform then l also scales as $O(L^2)$ and the average energy density is *independent* of the system size. Suppose, however, that the density of walls is hierarchical or fractal. Namely,

$$l \sim L^D. \tag{4.3}$$

The number D is the fractal dimension of the domain boundaries and must take a value between 1 and 2, $1 < D < 2$. The average energy density of the system is then

$$\epsilon \sim L^{D-2}. \tag{4.4}$$

This energy density *decreases* with the system size and becomes very small for macroscopic sizes. It cannot vanish altogether because the boundary conditions enforce existence of some domain walls, thus preventing the energy from falling completely to the ground state. Consider now what happens in realistic situations, where the system size is fixed at a given value. The question is then what pattern will the domains assume upon formation via a dynamic process. From the preceding discussion we expect that the domains will organize in a structure that *minimizes* the energy density. Under the assumption that any domain forming process acts on energy minimization principle, the above argument suggests that the domains will arrange in a hierarchical structure since this gives the lowest energy. Of course, there is always some uniform distribution of *large* domains that can give the same value for ϵ. I argue that the formation of such a distribution is unlikely. The reason is that for such a distribution to form one needs an information correlation length that is of order of the system size. In other words, local regions in the system need to 'know' that in far away regions the domain size is the right one. During the dynamic processes the system is far from equilibrium and magnetically disordered. It follows that the flow of information (e.g., in the form of spin density waves or any other dynamics that are the solution of Eq. (2.3)) is hindered by localization and damping. This, in turn, restricts the flow of information in the system and the information correlation length is much smaller than the system size. Thus it would be awfully difficult to achieve a global uniform distribution. Therefore, the system has to settle in one of the many (most probably even metastable) hierarchical structures. The exact pattern that the domain structure will assume cannot be predicted from the analysis presented here. Rather, it will be determined by the specific process that governs the formation dynamics.

5 CONCLUSION

To summarize, I have addressed formation and spontaneous arrangement of magnetic domains in two-dimensional ferromagnetic Heisenberg systems. The functional form of these solutions was found exactly and their nature has been discussed. I have shown that the magnetic energy in the system is concentrated along the domain boundaries and is exponentially small away from them. I argued that this suggests that minimization of the energy in the system is equivalent to minimization of the domain walls length and went on to predict that the pattern that the domain walls will assume is hierarchical or fractal due to energetic and dynamic considerations. Such structual arrangement can be made visible by magnetic force microscopy, as indeed seems to have been recently observed by Wuttig and coworkers [4]. There are a couple of points that need to be mentioned:

(i) There emerges some evidence [2] that the solutions discussed here persist even when there is coupling between the magnetic and elastic properties in the layer (as in magneto-elastic thin films). Depending on the coupling strengths, the occurrence of magnetic domain walls in such systems may lead to deformations, resulting in structural kinks that may be visible through traditional atomic force microscopy.

(ii) It has been shown recently [2] that the sharp domain wall solutions survive even if the system is made nonplanar, namely, when the plane is wrapped around a cylinder, or distorted into any other topology (torus, sphere). This suggests that many real–world magnetic systems can be used to test these predictions. For example, it would be very interesting to carry out an experiment on nanotubes. We conjecture that if nanotubes can be made magnetic then it is possible that such a domain structure, whether hierarchical or not, may be observed. Furthermore, if the nanotubes can be made magneto-elastic it may be possible to observe structural kinks on the cylinders. This, in turn, can have an enormous impact on magneto-mechanical control of these exotic systems, an issue that is currently being explored.

Finally, it should be emphasized that the solutions discussed here are different from the regular instanton solutions that usually occur when the BPE are involved. The reason is that in our system the boundary conditions are arbitrary, while in the traditional systems that follow the BPE the boundary conditions have to be uniform, thus restricting the possible solutions and eliminating the ones discussed here.

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References

[1] Balakrishnan, R. and R. Blumenfeld "Transformation of general curve evolution to a modified Belavin–Polyakov equation", *J. Math. Phys.* **38**, 5878 (1997).
[2] Blumenfeld, R. and A. Saxena (unpublished).
[3] Belavin, A. and A.M. Polyakov *Pis'ma Zh. Eksp. Teor. Fiz.* **22**, 503 (1975) [*JETP Lett.* **22**, 245 (1975)].
[4] Wuttig, M. These proceedings.